# Rapidity factorization and the EIC

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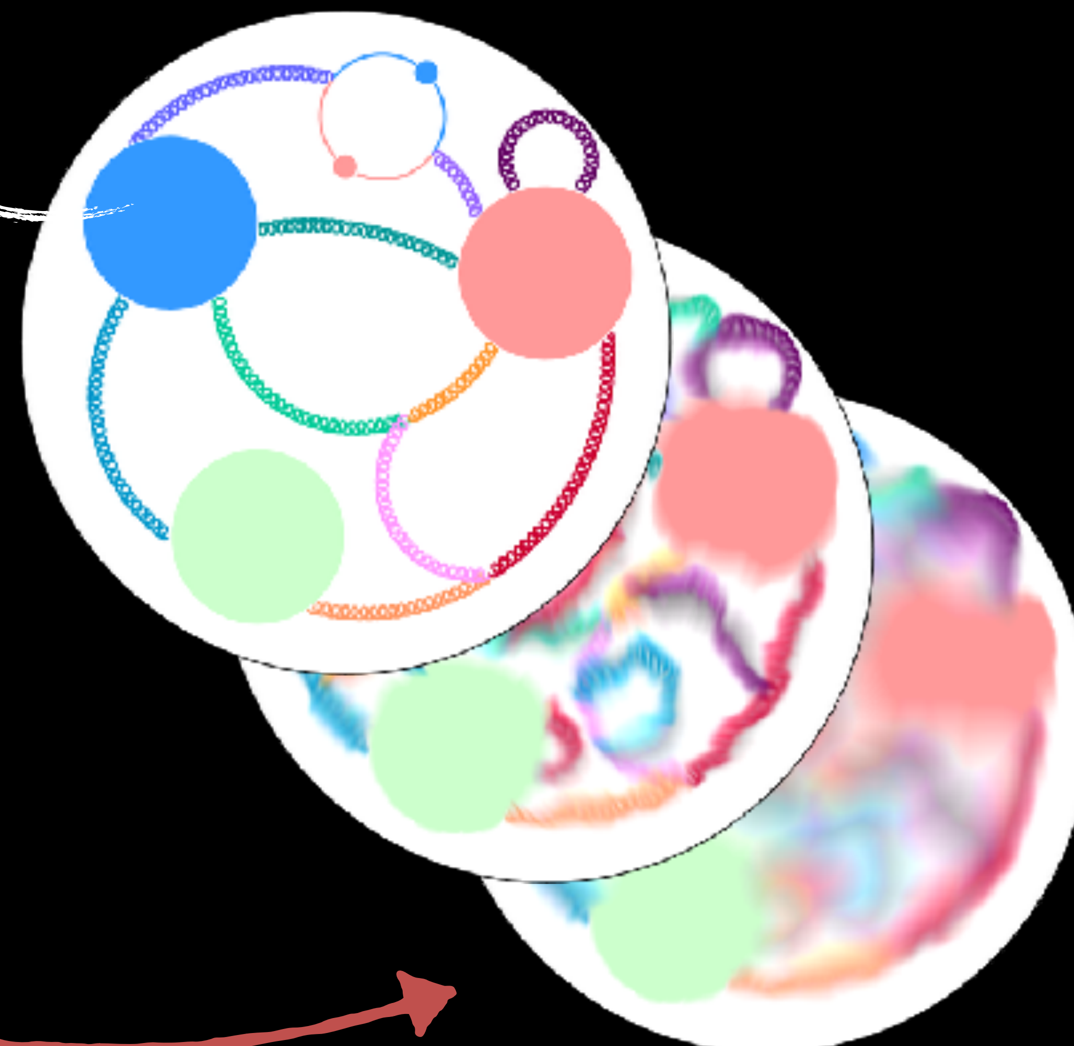
# Factorization

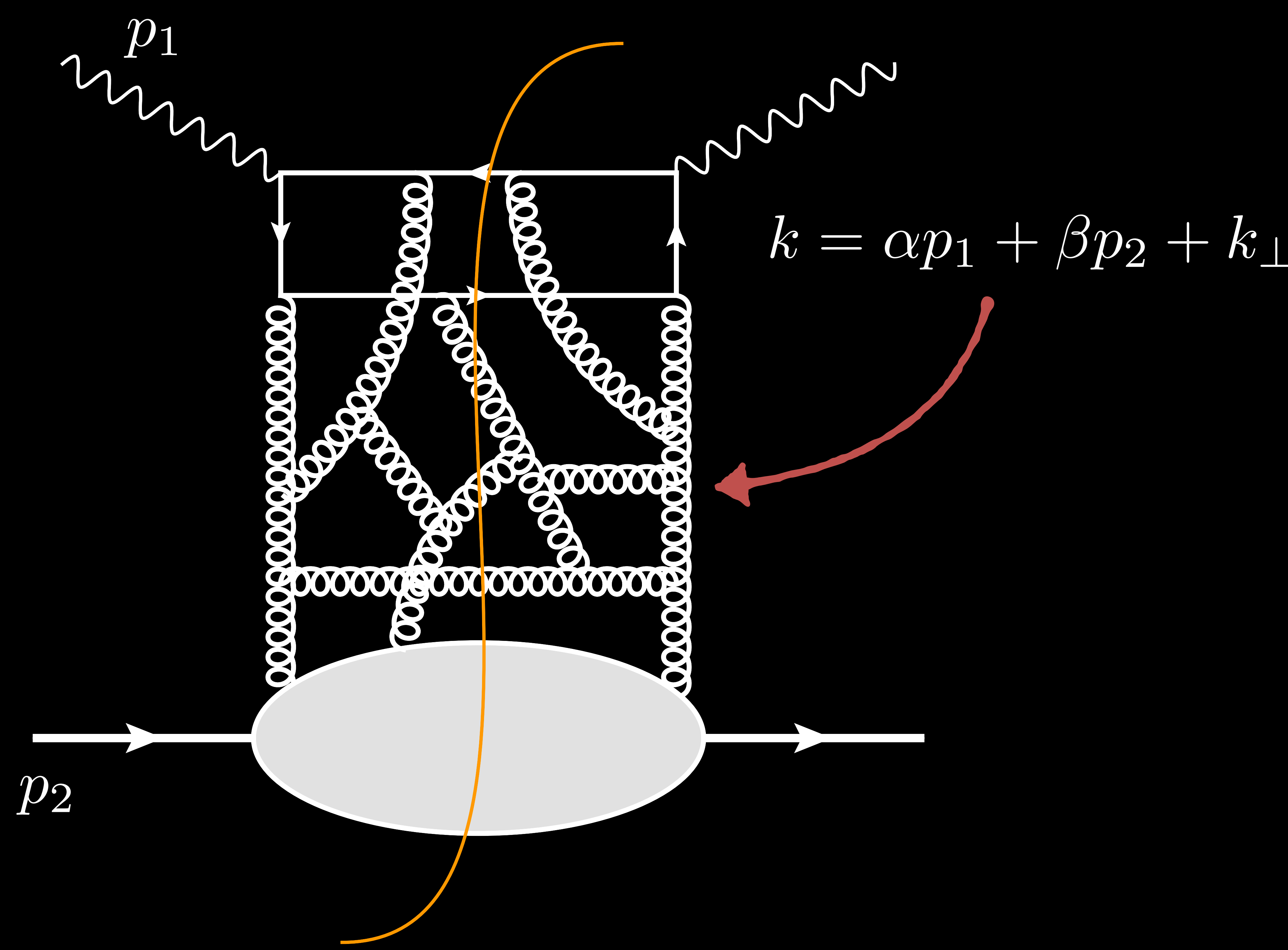
There are different types of factorization



From high-energy scattering reactions we can reconstruct the non-perturbative structure of the hadron

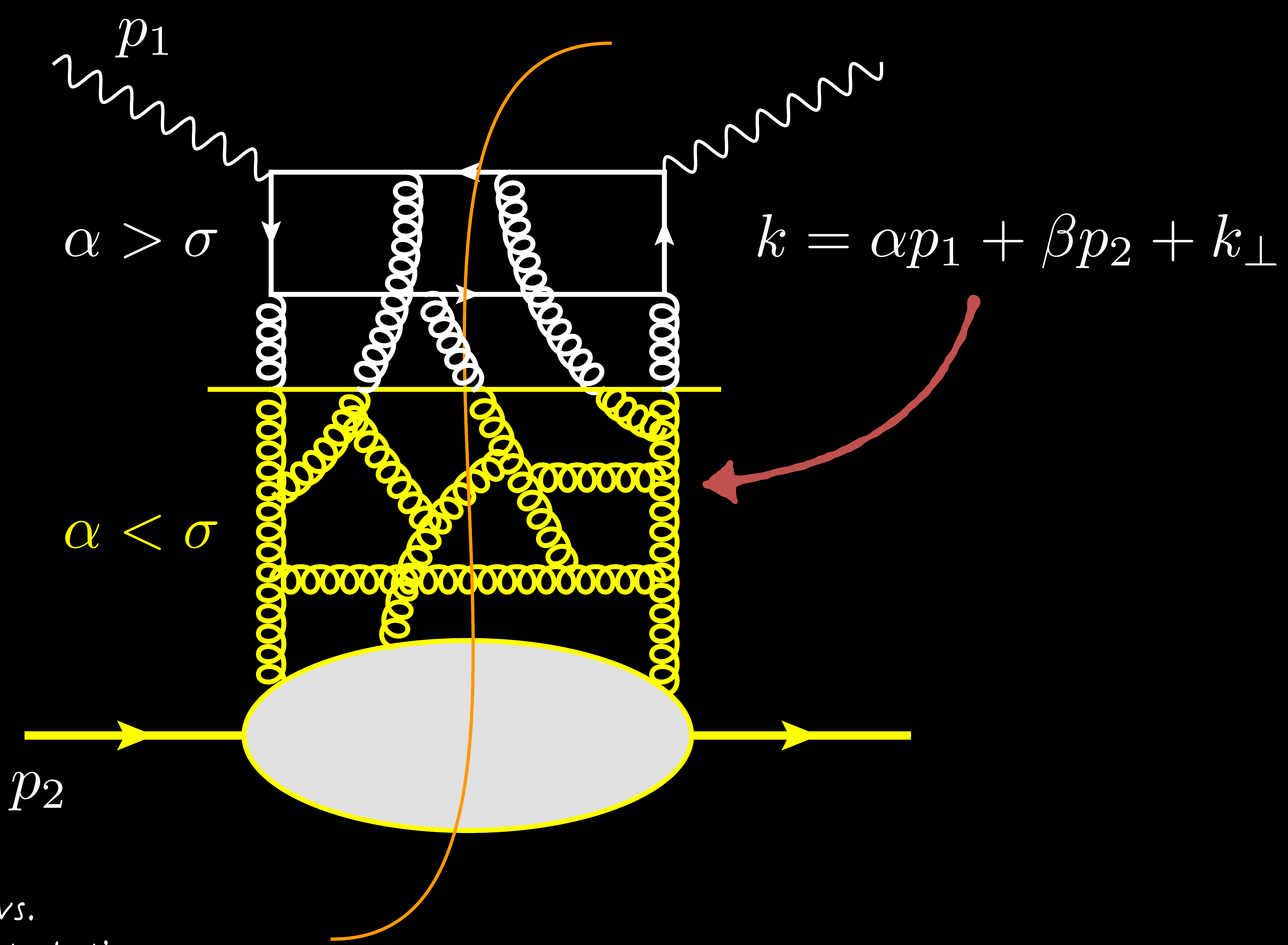
Layers are connected

# Rapidity factorization



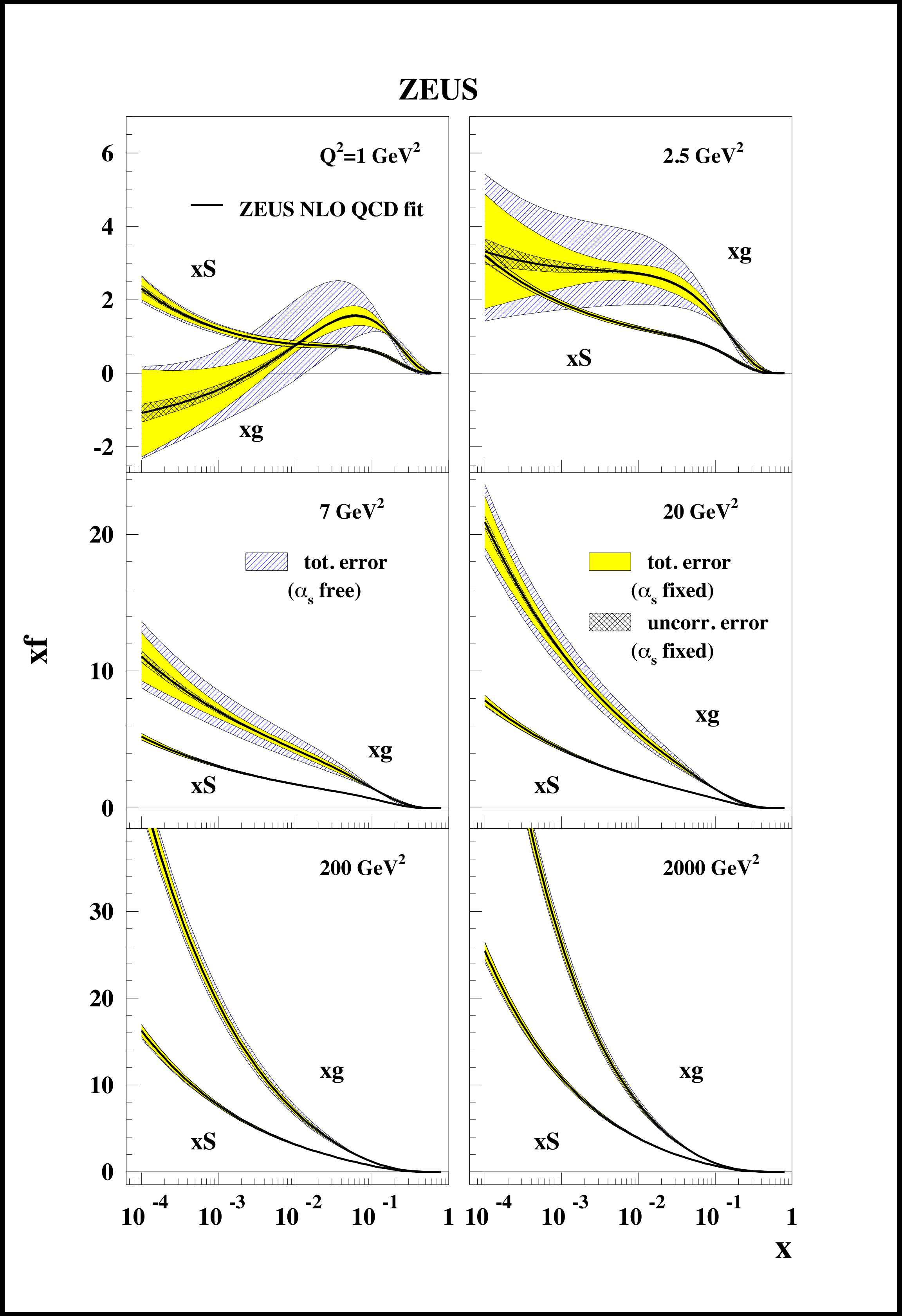
# Rapidity factorization

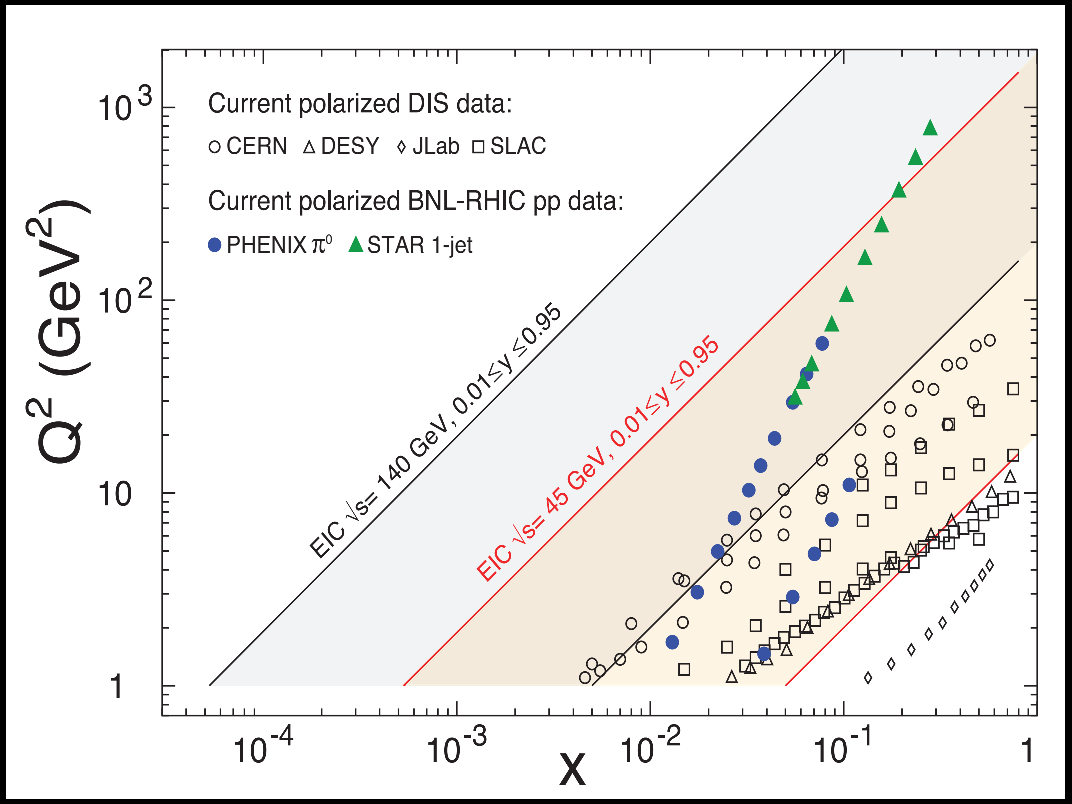
Separation of kinematic modes




Separation of modes vs. perturbative/non-perturbative

# PDFs at small-x



# Gluon TMDs

$$\mathcal{F}_i^a(\beta_B, x_\perp) \equiv \frac{2}{s}\int dx_*\; e^{i\beta_B x_*}[\infty, x_*]_x^{am} gF^m_{\bullet i}(x_*, x_\perp)$$

$\beta_B$

$$\tilde{\mathcal{F}}_j^a(\beta_B, y_\perp) \equiv \frac{2}{s}\int dy_*\; e^{-i\beta_B y_*} g\tilde{F}^m_{\bullet j}(y_*, y_\perp)[y_*, \infty]_y^{ma}$$

1. Balitsky & AT, JHEP 10 (2015) 017

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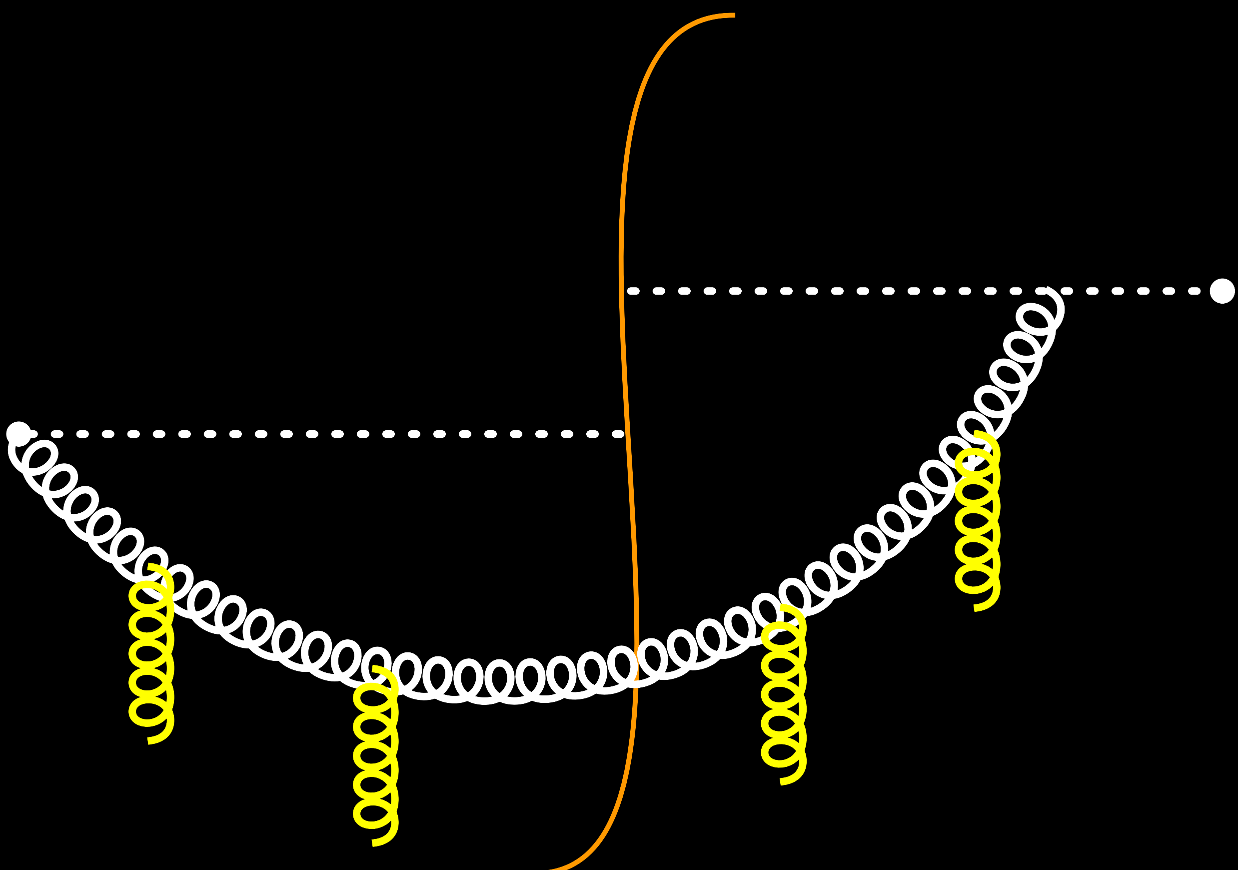
# Evolution of gluon TMDs

Different terms of expansion are important at small and moderate x



Generates Wilson-line structures

$$i\langle A^a_\mu(x)A^b_\nu(y)\rangle = (x|\frac{1}{\mathcal{P}^2+2i\mathcal{F}+i\epsilon}|y)^{ab}_{\mu\nu}$$

$$= -\frac{i}{2\pi}\int_{\sigma'}^{\sigma}\frac{d\alpha}{2\alpha}e^{-i\alpha(x-y)_\bullet}(x_\perp|P\exp\Big\{-i\int_{y_*}^{x_*}dz_*\Big[\frac{p_\perp^2}{\alpha s}-\frac{2}{s}A_\bullet(z_*)-\frac{2ig}{\alpha s}\mathcal{F}(z_*)\Big]\Big\}|y_\perp)^{ab}_{\mu\nu}$$

Explicit dependence on the cut-off

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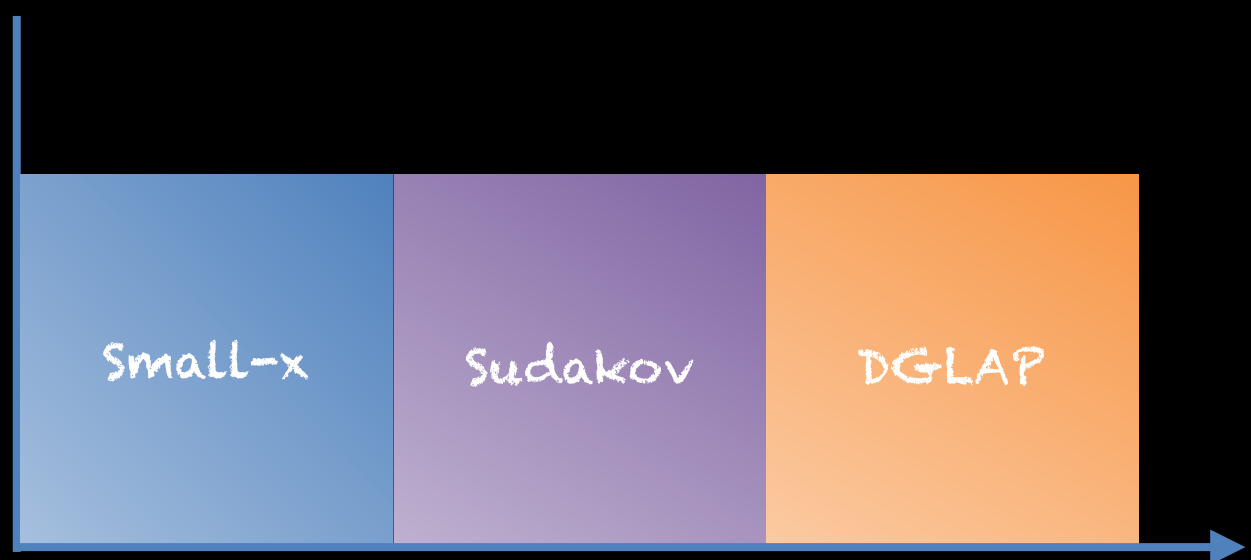
# Evolution equation for gluon TMDs

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\langle p|\mathcal{F}_i^a(x_B,x_\perp)\mathcal{F}_j^a(x_B,y_\perp)|p\rangle \\
&= -\alpha_s \mathrm{Tr}\Big\{\langle p|\int d\!\!{}^-{}^2k_\perp\; L_i{}^\mu(k,x_\perp,x_B)^{\mathrm{light-like}}\theta\Big(1-x_B-\frac{k_\perp^2}{\sigma s}\Big)L_{\mu j}(k,y_\perp,x_B)^{\mathrm{light-like}} \\
&+ 2\mathcal{F}_i(x_B,x_\perp)(y_\perp| - \frac{p^m}{p_\perp^2}\mathcal{F}_k(x_B)(i\overleftarrow{\partial}_l + U_l)(2\delta_m^k\delta_j^l - g_{jm}g^{kl})U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger \\
&+ \mathcal{F}_j(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|y_\perp) \\
&+ 2(x_\perp|U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger(2\delta_i^k\delta_m^l - g_{im}g^{kl})(i\partial_k - U_k)\mathcal{F}_l(x_B)\frac{p^m}{p_\perp^2} \\
&+ \mathcal{F}_i(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|x_\perp)\mathcal{F}_j(x_B,y_\perp)|p\rangle\Big\} + O(\alpha_s^2)
\end{aligned}
$$

I. Balitsky & AT, JHEP 10 (2015) 017

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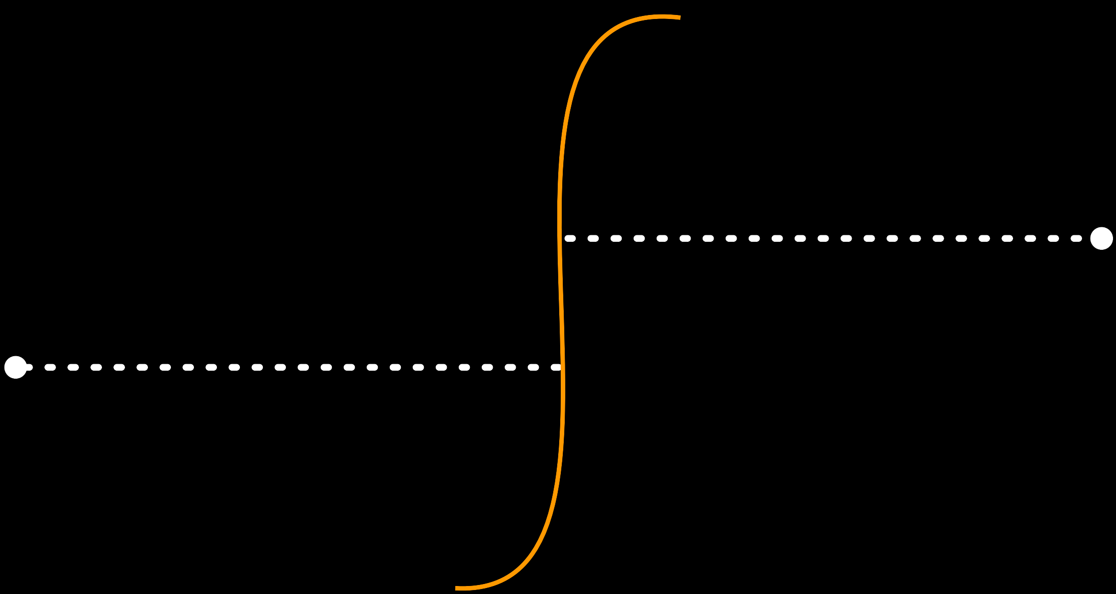
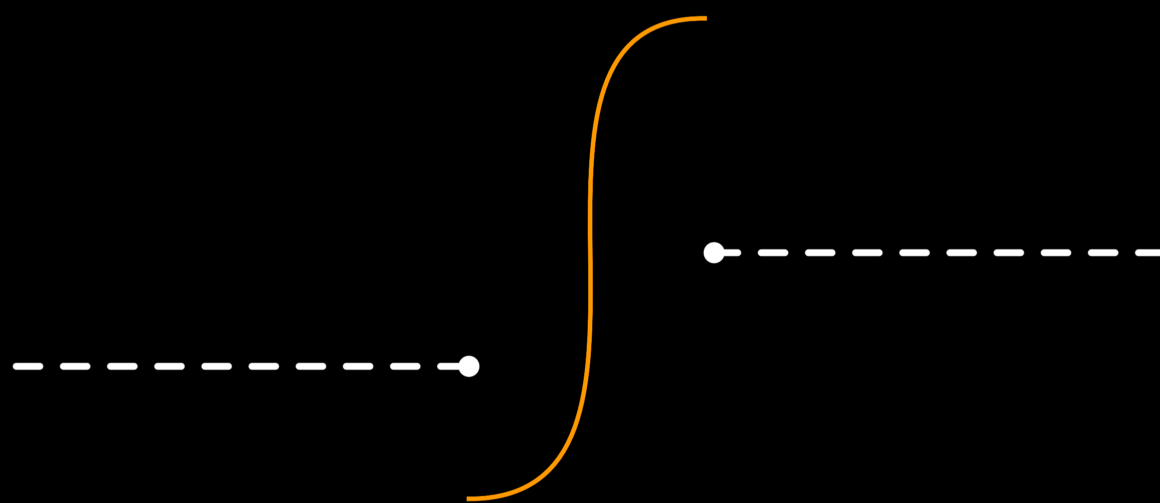
# Two TMD operators

Distribution function
(Drell-Yan)

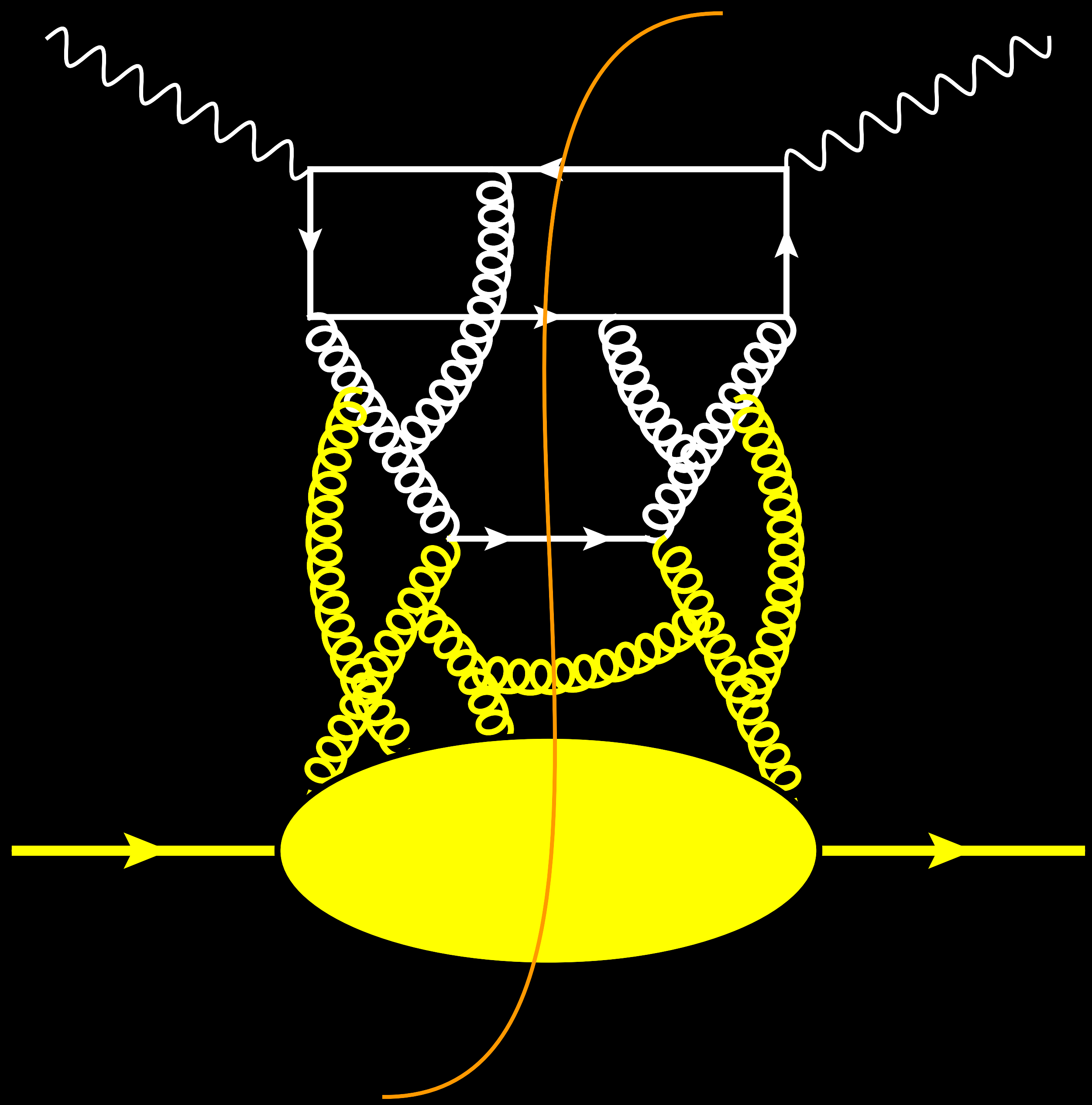




Fragmentation function

I. Balitsky & AT, JHEP 06 (2016) 164

# Particle production at EIC



$$\sigma_{\mu\nu} \propto \int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}DA$$

$$\sum_X \langle p|\tilde{T}\{j_\mu(w)F^2(x)\}|X\rangle_{\tilde{A}} \langle X|T\{F^2(y)j_\nu(0)\}|p\rangle_A$$

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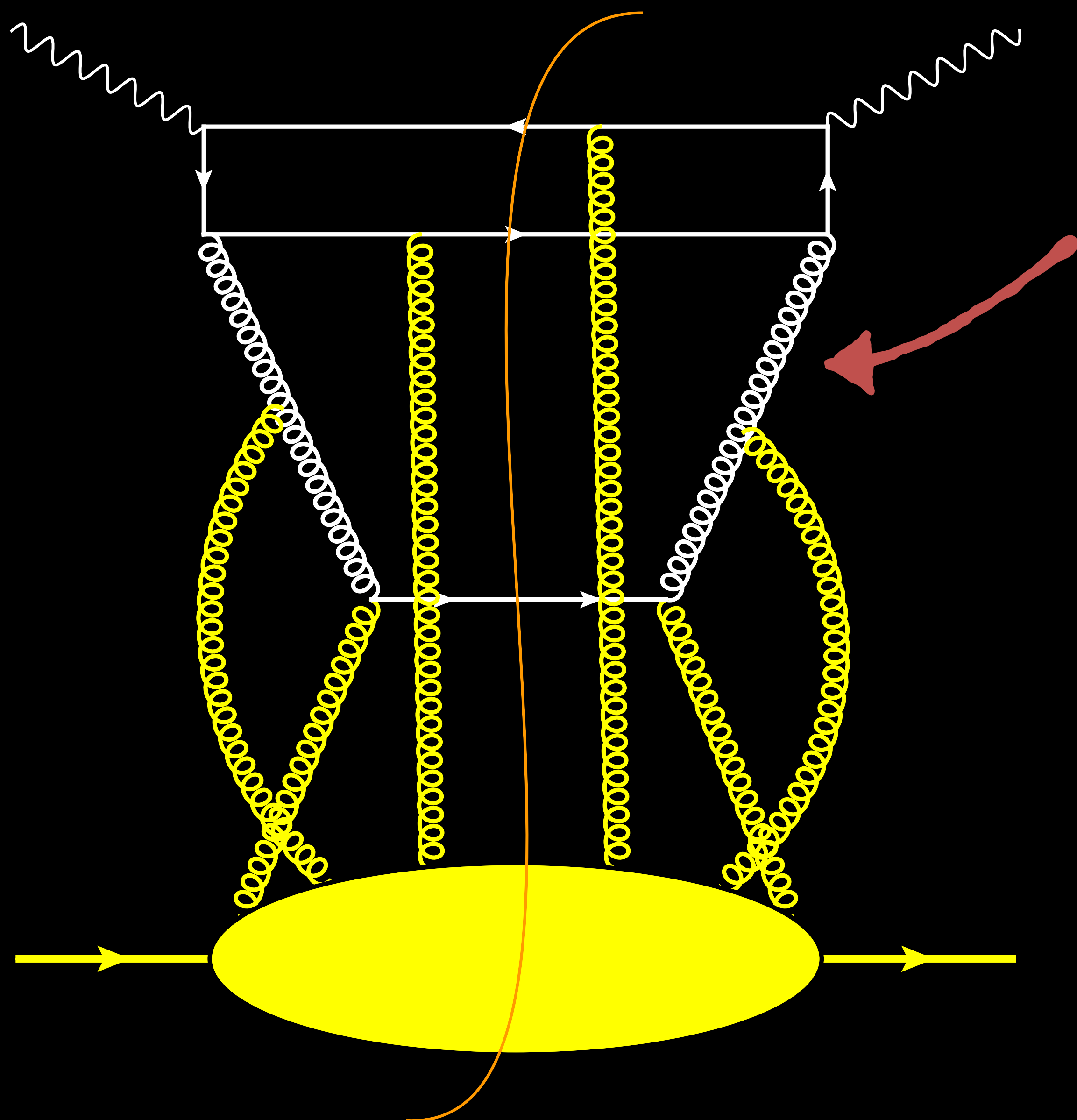
# Particle production. Color structure



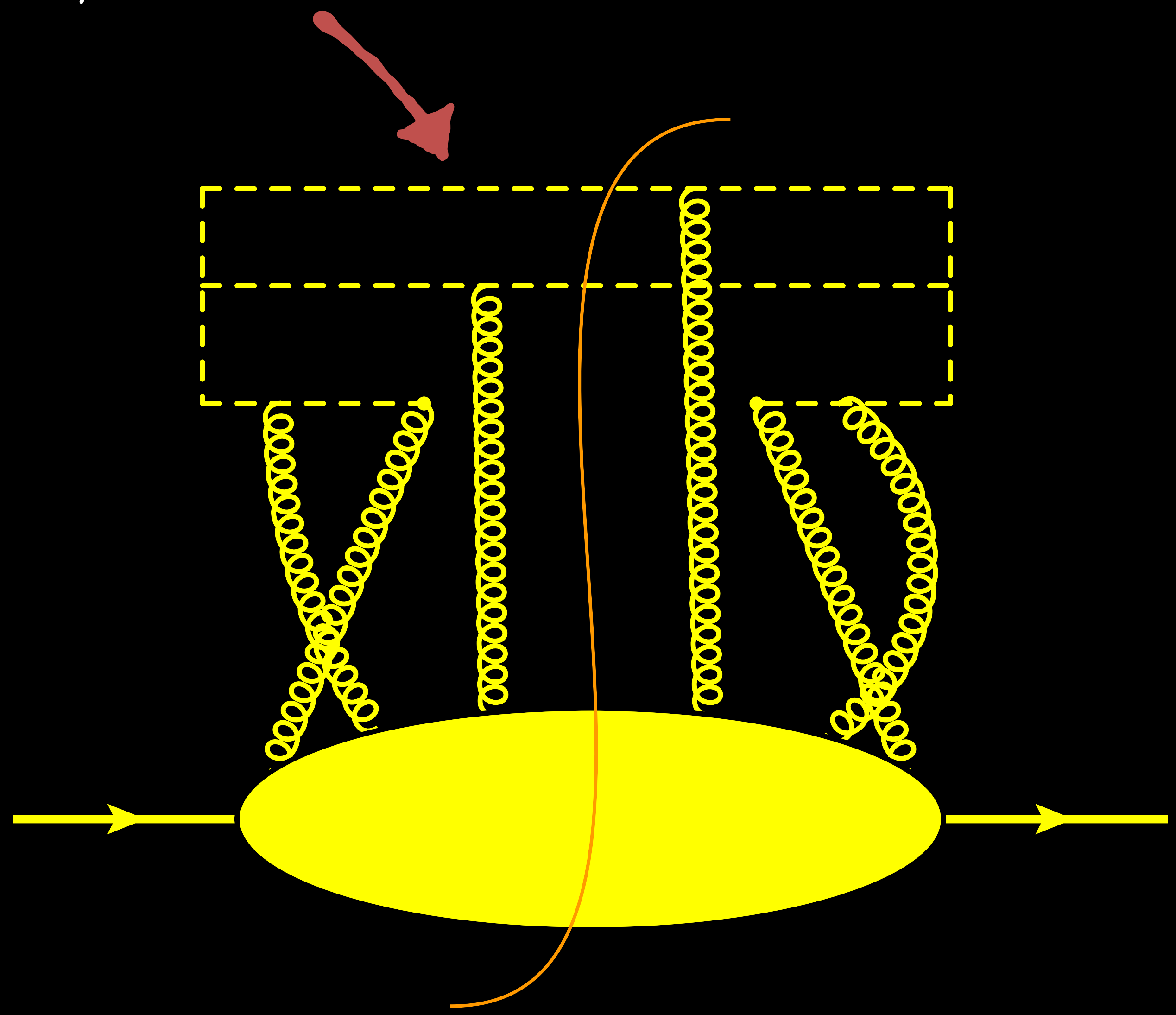
$$\langle A_\mu(z) F_{*j}(y)\rangle = \frac{i}{2}\int_\sigma^\infty \frac{đ\alpha}{2\alpha} e^{-i\alpha(y-z)_\bullet}$$

$$\times\ (z_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(y-z)_*}(\alpha s g^\perp_{\mu j} + 2p_{2\mu}p_j)|y_\perp)[-\infty, y_*]_y$$

Interaction of the target and projectile defines the structure of Wilson lines

# Particle production. Color structure

Light-like distance. We can get rid of this Wilson lines

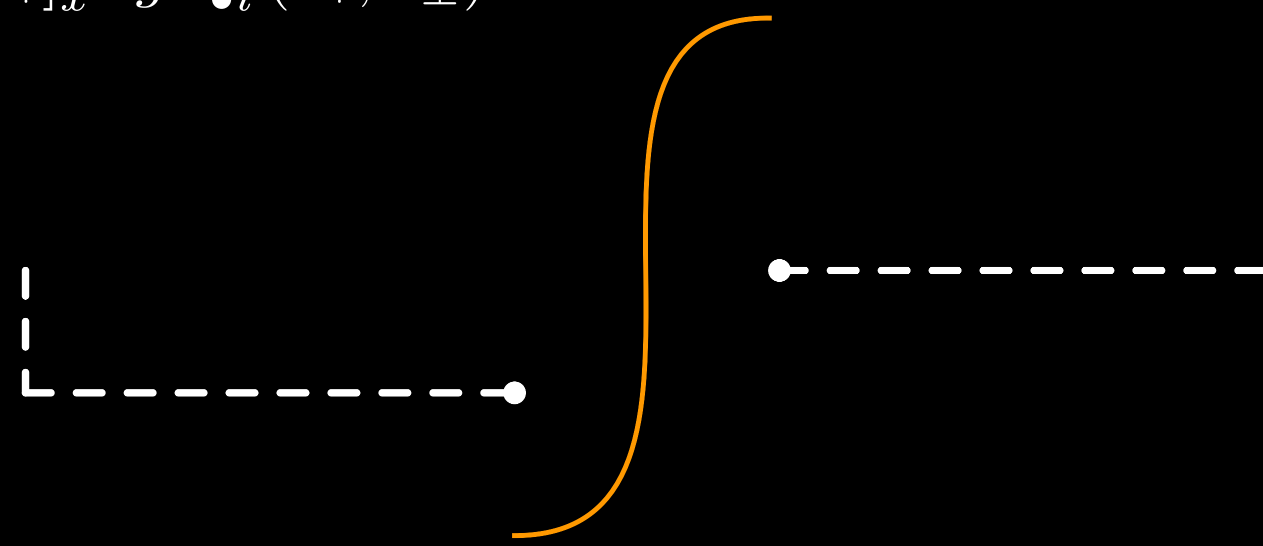


The color structure defines the TMD operator

Problem 1: The structure is not universal

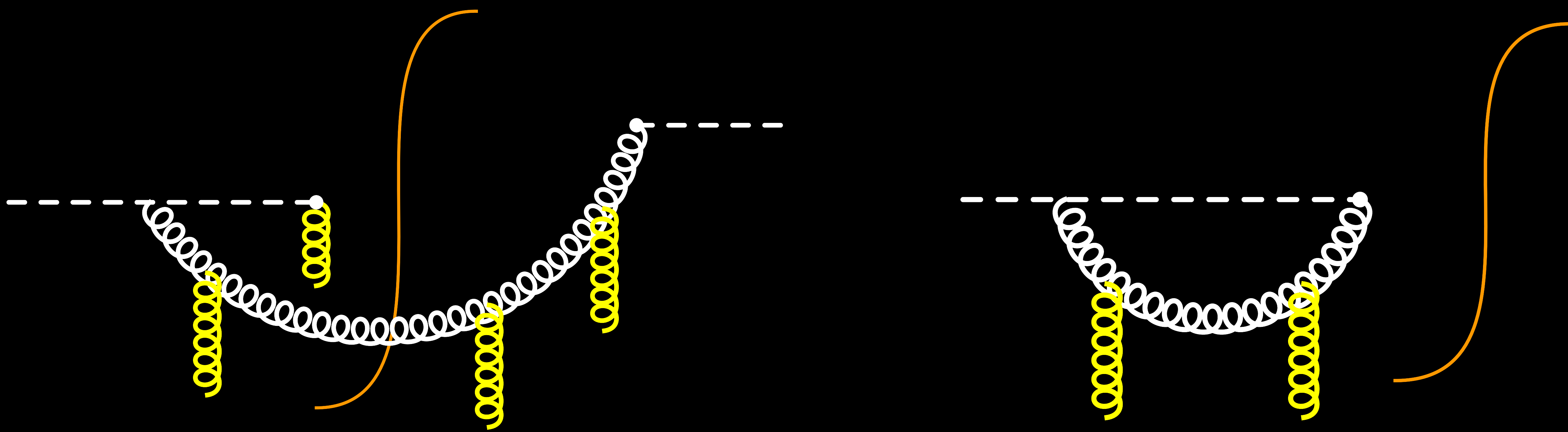
# TMD operator in Drell-Yan

$$\mathcal{F}_i^a(x_B,x_\perp) \equiv \frac{2}{s}\int dx_*\; e^{ix_Bx_*}[-\infty,x_*]_x^{am}\, gF^m_{\bullet i}(x_*,x_\perp)$$



$$\tilde{\mathcal{F}}_j^a(x_B,y_\perp) \equiv \frac{2}{s}\int dy_*\; e^{-ix_By_*}\, g\tilde{F}^m_{\bullet j}(y_*,y_\perp)[y_*,-\infty]_y^{ma}$$

# Gluon TMD evolution



The one loop correction has been calculated in the axial gauge

# Real emission

The results coincide up to change of direction in the color structures

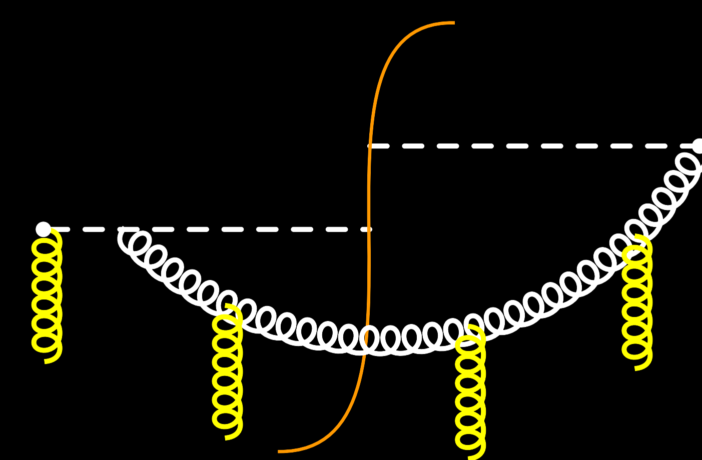
$$\frac{d}{d\ln\sigma}\tilde{\mathcal F}_i^{+a}(\beta_B,x_\perp)\mathcal F_j^{+a}(\beta_B,y_\perp)^{real}$$

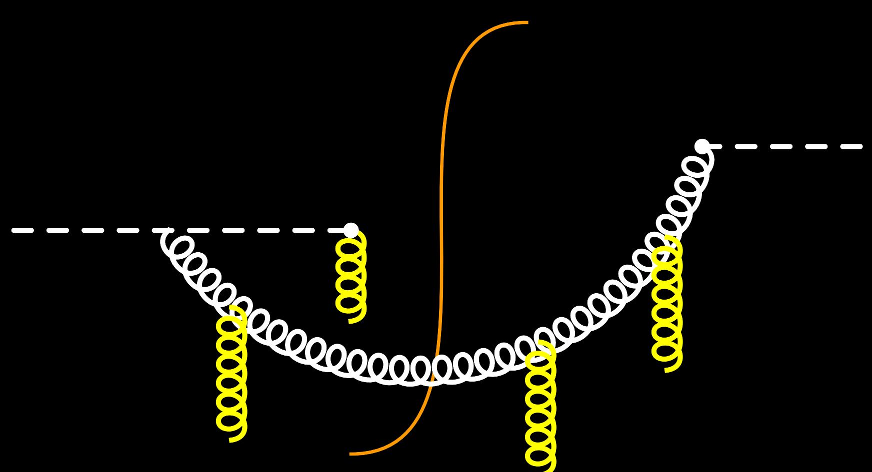
$$= -\alpha_s\mathrm{Tr}\Big\{\int \bar{d}^2k_\perp(x_\perp|\Big\{\tilde U\frac{1}{\sigma\beta_Bs+p_\perp^2}(\tilde U^\dagger k_k+p_k\tilde U^\dagger)\frac{\sigma\beta_Bsg_{\mu i}-2k_\mu^\perp k_i}{\sigma\beta_Bs+k_\perp^2}$$

$$-\ 2k_\mu^\perp g_{ik}\tilde U\frac{1}{\sigma\beta_Bs+p_\perp^2}\tilde U^\dagger-2g_{\mu k}\tilde U\frac{p_i}{\sigma\beta_Bs+p_\perp^2}\tilde U^\dagger+\frac{2k_\mu^\perp}{k_\perp^2}g_{ik}\Big\}\tilde{\mathcal F}^{+k}\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|k_\perp)$$

$$\times\ (k_\perp|\mathcal F^{+l}\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)\Big\{\frac{\sigma\beta_Bs\delta_j^\mu-2k_\perp^\mu k_j}{\sigma\beta_Bs+k_\perp^2}(k_lU+Up_l)\frac{1}{\sigma\beta_Bs+p_\perp^2}U^\dagger$$

$$-\ 2k_\perp^\mu g_{jl}U\frac{1}{\sigma\beta_Bs+p_\perp^2}U^\dagger-\ 2\delta_l^\mu U\frac{p_j}{\sigma\beta_Bs+p_\perp^2}U^\dagger+2g_{jl}\frac{k_\perp^\mu}{k_\perp^2}\Big\}|y_\perp)$$





$$\frac{d}{d\ln\sigma}\tilde{\mathcal F}_i^{a-}(\beta_B,x_\perp)\mathcal F_j^{-a}(\beta_B,y_\perp)$$

$$= -\alpha_s\mathrm{Tr}\Big\{\int \bar{d}^2k_\perp(x_\perp|\Big\{U^\dagger\frac{1}{\sigma\beta_Bs+p_\perp^2}(Uk_k+p_kU)\frac{\sigma\beta_Bsg_{\mu i}-2k_\mu^\perp k_i}{\sigma\beta_Bs+k_\perp^2}$$

$$-\ 2k_\mu^\perp g_{ik}U^\dagger\frac{1}{\sigma\beta_Bs+p_\perp^2}U-2g_{\mu k}U^\dagger\frac{p_i}{\sigma\beta_Bs+p_\perp^2}U+\frac{2k_\mu^\perp}{k_\perp^2}g_{ik}\Big\}\tilde{\mathcal F}^{-k}\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|k_\perp)$$

$$\times\ (k_\perp|\mathcal F^{-l}\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)\Big\{\frac{\sigma\beta_Bs\delta_j^\mu-2k_\perp^\mu k_j}{\sigma\beta_Bs+k_\perp^2}(k_lU^\dagger+U^\dagger p_l)\frac{1}{\sigma\beta_Bs+p_\perp^2}U$$

$$-\ 2k_\perp^\mu g_{jl}U^\dagger\frac{1}{\sigma\beta_Bs+p_\perp^2}U-\ 2\delta_l^\mu U^\dagger\frac{p_j}{\sigma\beta_Bs+p_\perp^2}U+2g_{jl}\frac{k_\perp^\mu}{k_\perp^2}\Big\}|y_\perp)$$

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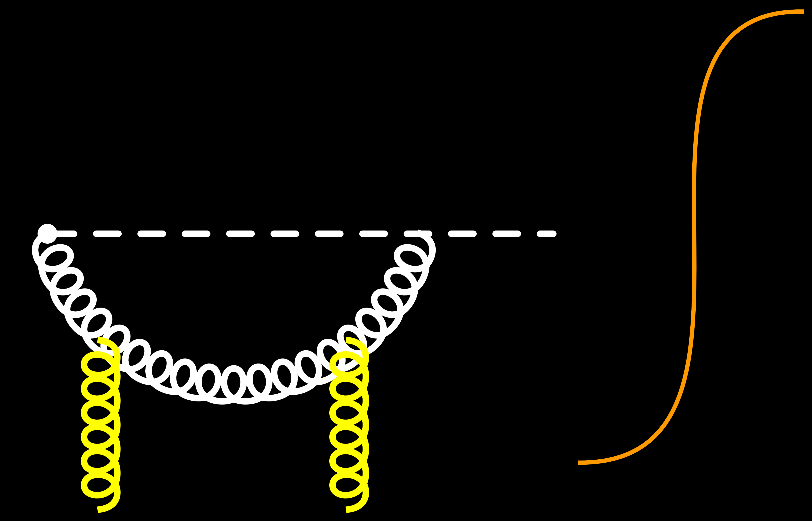
# Virtual emission

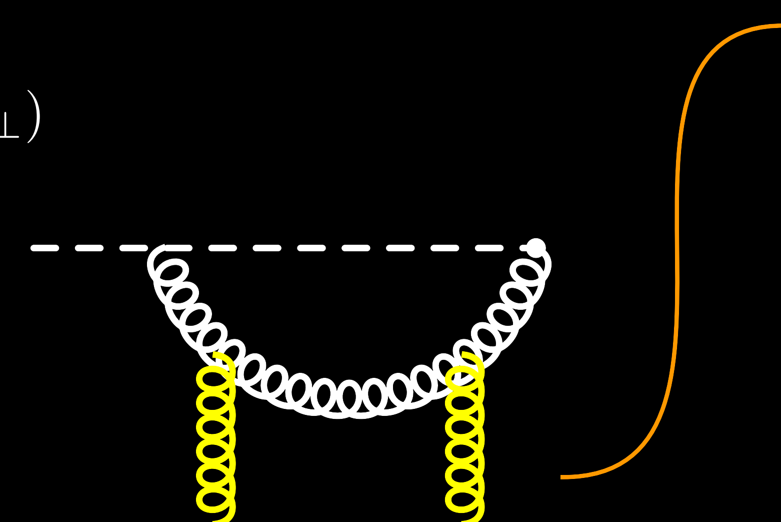
$$\mathcal{F}_i^{+a}(\beta_B, y_\perp)^{virt} = \; - N_c \mathcal{F}_i^{+a}(\beta_B, y_\perp) \int_{\sigma'}^{\sigma} \frac{d\!\!\bar{}\,\alpha}{\alpha} (y_\perp | \frac{\alpha\beta_B s}{p_\perp^2(\alpha\beta_B s + p_\perp^2)} | y_\perp)$$
$$- \int_{\sigma'}^{\sigma} \frac{d\!\!\bar{}\,\alpha}{\alpha} TrT^a (y_\perp | \frac{p_j}{p_\perp^2} (\delta_i^j \partial_\perp^2 U + 2\partial_i \partial^j U) \frac{1}{\alpha\beta_B s + p_\perp^2} U^\dagger | y_\perp)$$



Problem 2: Different structure of singularities

Can we observe it in evolution?

$$\mathcal{F}_i^{-a}(\beta_B, y_\perp)^{virt} \; = \; - \; N_c \mathcal{F}_i^{-a}(\beta_B, y_\perp) \int_{\sigma'}^{\sigma} \frac{d\!\!\bar{}\,\alpha}{\alpha} (y_\perp | \frac{\alpha\beta_B s}{p_\perp^2(\alpha\beta_B s - p_\perp^2 + i\epsilon)} | y_\perp)$$
$$- \int_{\sigma'}^{\sigma} \frac{d\!\!\bar{}\,\alpha}{\alpha} \mathrm{Tr} T^a U_y^\dagger (y_\perp | \frac{1}{\alpha\beta_B s - p_\perp^2 + i\epsilon} (\delta_i^j \partial_\perp^2 U + 2\partial_i \partial^j U) \frac{p_j}{p_\perp^2} | y_\perp)$$



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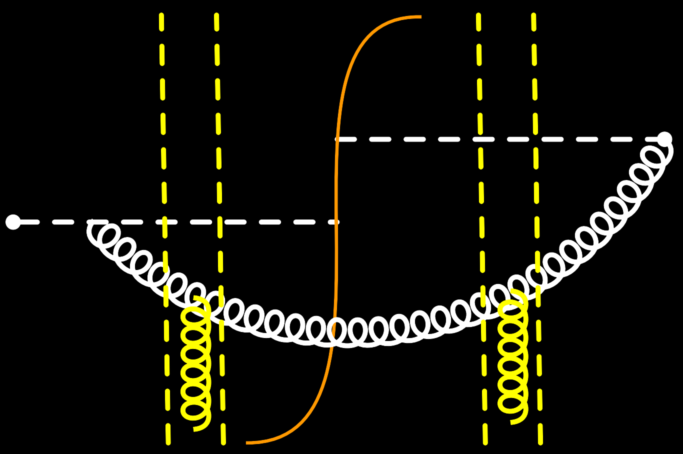
# Small-x evolution

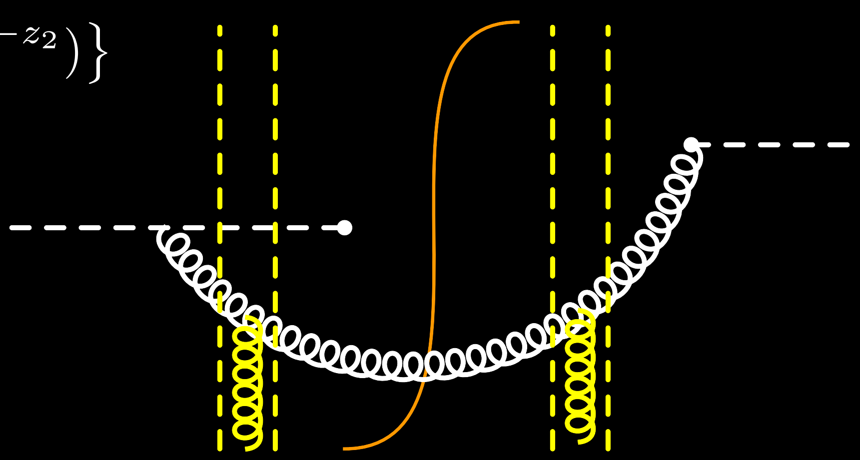


$$\frac{d}{d\eta}U_i^{+a}(z_1)U_j^{+a}(z_2)$$
$$= -\frac{g^2}{8\pi^3}\mathrm{Tr}\{(-i\partial_i^{z_1}+U_i^{+z_1})[\int d^2z_3(U_{z_1}U_{z_3}^\dagger-1)\frac{z_{12}^2}{z_{13}^2z_{23}^2}(U_{z_3}U_{z_2}^\dagger-1)](i\overleftarrow{\partial}_j^{z_2}+U_j^{+z_2})\}$$

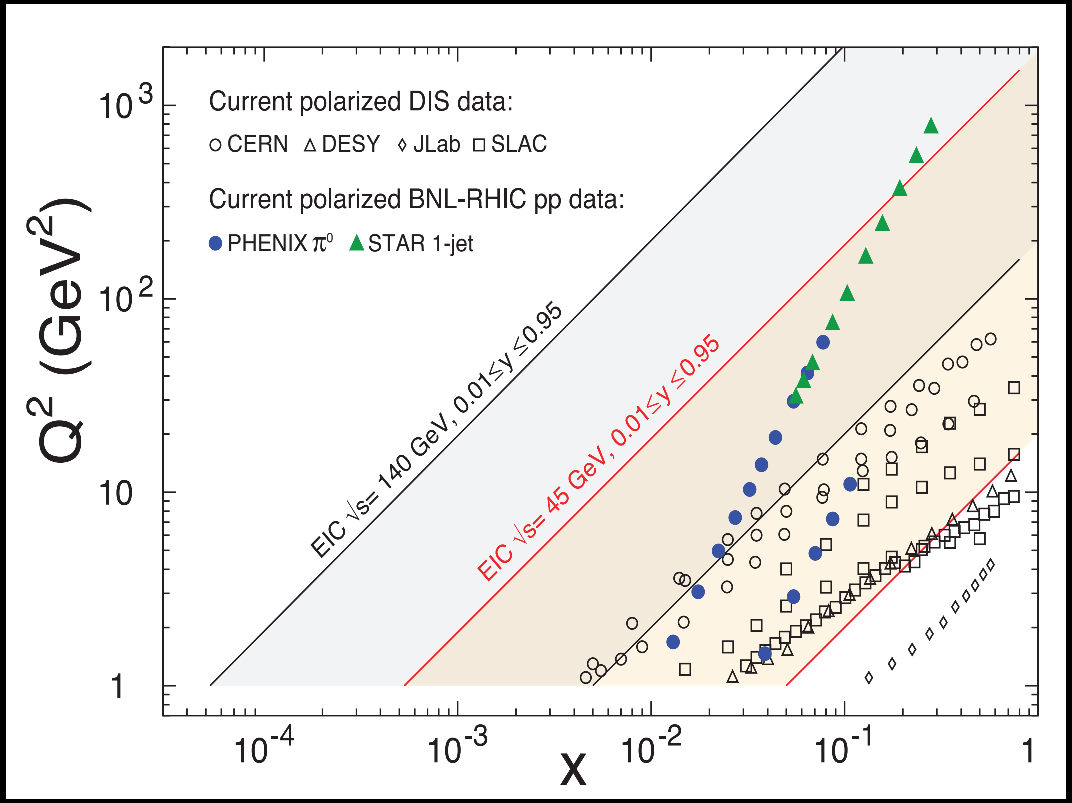
In interesting kinematic (small-x, large-x) there is no difference in evolution of the TMD operator

However. . .

$$\frac{d}{d\eta}U_i^{-a}(z_1)U_j^{-a}(z_2)$$
$$= -\frac{g^2}{8\pi^3}\mathrm{Tr}\{(i\partial_i^{z_1}+U_i^{-z_1})[\int d^2z_3(U_{z_1}^\dagger U_{z_3}-1)\frac{z_{12}^2}{z_{13}^2z_{23}^2}(U_{z_3}^\dagger U_{z_2}-1)](-i\overleftarrow{\partial}_j^{z_2}+U_j^{-z_2})\}$$



# Intermediate region




Can we observe this pole in the intermediate region?

$$\mathcal{F}_i^{-a}(\beta_B, y_\perp)^{virt} \; = \; - \; N_c \mathcal{F}_i^{-a}(\beta_B, y_\perp) \int_{\sigma'}^{\sigma} \frac{đ\alpha}{\alpha} (y_\perp | \frac{\alpha\beta_B s}{p_\perp^2(\alpha\beta_B s - p_\perp^2 + i\epsilon)} | y_\perp)$$

$$- \int_{\sigma'}^{\sigma} \frac{đ\alpha}{\alpha} \mathrm{Tr} T^a U_y^\dagger (y_\perp | \frac{1}{\alpha\beta_B s - p_\perp^2 + i\epsilon} (\delta_i^j \partial_\perp^2 U + 2\partial_i \partial^j U) \frac{p_j}{p_\perp^2} | y_\perp)$$